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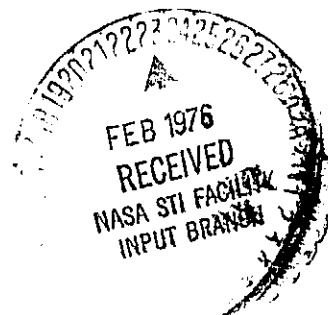
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Literature Review of Some Selected  
Types of Results and Statistical Analyses of  
Total-Ozone Data

January, 1976



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## I. Introduction

Recently in the news there has been much publicity concerning the possible depletion of ozone in the stratosphere. The Concorde supersonic aircraft is now flying transatlantic routes and the French government has petitioned to obtain permission to land in the U.S. Large quantities of exhaust effluents from these aircraft as well as other pollutants from industrial operations will be injected into the stratosphere at all altitudes. The  $\text{NO}_x$  compounds found in these pollutants are a potentially serious problem because they are a part of a chemical process that destroys ozone. And fluorocarbons used in the manufacture of spray can deodorants, for instance, have been cited as another possible ozone-destroying compound. Since the stratosphere is thermally stable and in radiative balance, particles and gases, such as the  $\text{NO}_x$  compounds or aerosols, have residence times measured in years rather than days or weeks as in the troposphere. The danger of injecting ozone-destroying compounds into the stratosphere becomes apparent when considering the function of ozone.

Ozone, a tri-atomic allotrope of oxygen, occupies only about  $4 \times 10^{-7}$  by volume of a column through the atmosphere and is therefore classified as one of the minor constituents, but because of its role in the maintenance of life on earth, it is equal in importance to the major constituents. Ozone, which is created in the stratosphere, shields life on earth from the solar ultraviolet radiation at wavelengths shorter than  $3000^\circ \text{\AA}$ . It also plays an important role in the radiation and heat budget of the stratosphere; and, therefore, has a major part in the earth's climatology. A small decrease in ozone concentration would produce an increase in the intensity of ultraviolet rays that reach the ground. This could result in climatic changes on the earth. The increase in ultraviolet can damage plants, by inhibition of

photosynthesis, mutation, or burning of growth tissue. Changes in the climate especially in temperature and precipitation can have a large effect on plants, especially crops that may be necessary for survival in marginal growing areas. Also, a small reduction in ozone may lead to an increased incidence of skin cancer, especially in the tropical and more temperate climates like the United States. It has been shown in Urbach, Berger, and Davis [27] that, based on conservative estimates, for the U.S. the eventual increase in skin cancer may be expected to range from 14.6 to 21/100,000 or 30,600 to 44,000 new cases of skin cancer per year, or an increase of 10 to 15% for a 5% decrease in ozone. If the ozone decrease was as much as 40%, the increase in skin cancer may be as high as 116.7 to 278/100,000 or 244,500 to 582,500 additional cases per year, or an increase of 81.5 to 194%. Another function of ozone is that it is useful as a tracer of the general atmospheric circulation, stratosphere-troposphere interchange, and in synoptic weather patterns, which is another area of importance to man.

Due to these growing concerns the ozone concentration in the stratosphere has been measured by ground stations, aircraft, satellites, balloons and rockets; and it is still being monitored daily.

## II. Total Ozone

Total ozone is the measure of the amount of ozone in a vertical column of the earth's atmosphere. This is a different concept from ozone partial pressure, or the vertical distribution of ozone, which is a function of the height in the atmosphere. Measurements of total ozone have been made since 1912. A theory was developed quite early which stated that the cut-off of solar ultraviolet radiation observed near the earth's surface was due to ozone absorption. Therefore, Dobson constructed an instrument using a

spectrographic technique to measure the total amount of ozone over a particular point. Dobson's spectrophotometer is still considered to provide highly accurate measurements of total ozone and therefore is the standard instrument for the measure of total ozone. Other types of instruments exist and are usually evaluated in comparison to Dobson's.

A network of stations for ground based measurements of total ozone have been developed throughout the world, and particularly in the Northern Hemisphere; thus many features of the global distribution of total ozone are available for study. The Dobson spectrophotometer provides the longest and most extensive series of observations. One of the major sources of ground based measurements of total ozone is the series of bimonthly publications entitled "Ozone Data for the World." Data since about 1960 has been tabulated and includes daily values and monthly mean values of total ozone for approximately 170 stations distributed over both hemispheres, although a large proportion are in the Northern Hemisphere. There are other sources, such as individual stations, that have earlier records of ground-based total-ozone measurements. Arosa, Switzerland, has records of total-ozone measurements dating back to 1926 and other stations have measurements dating from the 1920's and 1930's on; but Arosa (since 1932), and Oxford, U.K. (since 1951) are the only two stations that have continuous total-ozone observations for more than 20 years. Total-ozone data sufficient to determine a global distribution did not become available until 1957.

For ground-based measurements, the Dobson spectrophotometer is the most widely used instrument for the monitoring of stratospheric ozone. It is a quartz prism double monochromator which uses a double wavelength pair method with direct sunlight. It is considered to provide highly accurate measurements of total ozone, its results believed to be within  $\pm 1.5\%$ .

Krueger and Pressman [15] state that there is an empirical relationship between measurements on direct sunlight and zenith sunlight that allows for estimates of total ozone on days that are partly cloudy or cloudy. Dobson's spectrophotometer is often used as a standard for checking the accuracy of total-ozone measurements obtained from other instruments as well as satellites.

Brewer and Kerr [4] became concerned with total-ozone measurements under cloudy skies. A problem associated with ground-based measurements is the attainment of accurate values of total ozone in cloudy weather. This effect of cloud cover on total-ozone measurements has been considered using the Toronto spectrophotometer. There are two cloud effects on total-ozone measurements. One is a purely optical effect which shows up as an apparent increase in ozone. In addition to this effect due to measurement error there may be a real increase in ozone due to the presence of large cumulus-type clouds.

Another type of instrument used is a filter ozonometer type M-83. This is used at approximately  $1/3$  of the stations in the network system, mainly in the USSR and East Germany. Also, a few French stations use the Vassy type instrument.

Bojkov [2] used a comparison of statistical characteristics of total-ozone measurements made at stations equipped with Dobson spectrophotometers or with filter ozonometers M-83 to indicate the differences in the two type of instruments. Some of the stations show that the distance between total-ozone stations should not exceed 800-900 km., but more work is needed on the requirements for maximum effectiveness of the ozone network. Also, these studies indicate a strong dependence of filter ozonometer readings on the sun's position and visibility, which give filter ozonometer readings that vary two to five times as much as the Dobson readings. Readings of the

filter ozonometer have been observed to be 6% lower to 30% higher than the Dobson ozone readings. Simultaneous observations made with both instruments show poor correlation. This variability with the M-83 can be reduced if readings are made symmetrically around local noon and only when the sun is more than  $30^\circ$  but less than  $50^\circ$  above the horizon.

Another indication of differences in the measurements of these two types of instruments is found by Picha [22] looking at total ozone and the temperature field at 100 mb over Europe for 1967-1968. It has been found that a close positive relation exists between the amount of ozone and temperature of the stratospheric layer, but here the global distribution maps of ozone and temperature are contradictory. The fact that two different types of instruments are used in Europe to measure ozone seems to explain this discrepancy.

Dobson spectrophotometers are used in the western area and the Soviet filter instruments are used in the eastern region making the results not comparable. Therefore, in Picha's study the temperature field is included in constructing realistic global ozone distribution maps since the present ground-based total-ozone measurements are unsatisfactory.

To obtain total-ozone measurements to study the general circulation of the atmosphere, Mani [19] emphasizes the urgent need to establish adequate ozone-observing networks where the different types of instrumentation now being used are standardized and uniform calibration procedures are adopted. At best there is a 6% difference in measurement accuracy between these two main types of instruments, so to make the data comparable, comparisons should be used to obtain precise corrections to be applied to both types of instruments. Also, it is necessary to standardize the instruments among themselves so that observations among stations using one type of instrument



are comparable. A system of reference should be established to ensure a continuous set of comparable ozone measurements and to check the globally obtained satellite measurements.

Differences in instrumentation is only one of the factors to be considered before analyzing the total ozone data. There are other characteristics of the data and of the measuring network as a whole that should be considered. The present total ozone network is far from the ideal because ocean areas of the Northern Hemisphere are unrepresented and areas of the tropics and the Southern Hemisphere are under-represented. This may be improved by the use of satellite derived total-ozone data. It has been suggested by Pittcock [23] that the existing estimates of global ozone trends may be seriously biased by the distribution of the observing stations. Often these stations were chosen for historical, economic, and political reasons and are therefore not distributed uniformly. They are mainly located on the continents so there are few measurements for oceanic regions. We are getting a good characterization of total ozone for the individual stations, but they are often located at mid-latitudes which gives a poor characterization for latitude zones which are often used in an analysis. Many who have studied and tried to analyze total-ozone data feel that more data is needed at more stations. There is also a question as to the adequate length of record needed to establish global results concerning ozone.

Recently, data derived from satellites has begun to play a more important role in the study and analysis of ozone data. Satellite-derived maps of total ozone show a considerable amount of detailed information that could not be obtained from the Dobson, or ground-based, measuring stations which are limited to continental regions. Satellite-derived ozone measure-

ments provide a more adequate coverage of the earth as well as a more uniform coverage allowing more accurate measurements of zonal means and the global daily ozone budget.

One type of instrumentation that has been flown on Nimbus satellites is the Infrared Interferometer Spectrometer (IRIS). To assess the reliability of measurements using this instrument, the data was compared with the Dobson spectrophotometer total-ozone measurements over the Northern and Southern Hemispheres by Prabhakara, Rodgers, and Salomonson [24]. The two sets of data were found to agree within about 6% over mid-latitudes and not quite as good in the tropics.

Also, with the IRIS as well as other satellite measurements, it has been possible to deduce the global distribution of total ozone on a daily basis and monthly mean maps have been obtained. Using the ground based measurements, "Ozone Data for the World," one can get zonal means and daily measurements of total ozone, but these are not as accurate as those obtained from satellites because of the difference in measurement systems in the ground-based data and the distribution of the stations. One important limitation in the technique of using remote sensing of ozone from infrared data when using IRIS is when the atmosphere is nearly isothermal for then data measurements are not accurate.

Another type of instrument used on satellites, and in particular on Nimbus-4, is the Backscatter Ultraviolet (BUV) atmospheric ozone experiment. Krueger and Pressman [15] consider it the most successful experiment to date for obtaining ozone data. Heath, Mateer and Krueger [9] give a description of the BUV instrumentation and how the inference of total ozone is made from measurements at ultraviolet wavelengths which are weakly absorbed near the long-wavelength end of the ozone absorption band. Also, this BUV instrumentation has maintained a wavelength accuracy of better than 1 Å after

two years of operation on Nimbus-4. Comparing the Dobson spectrophotometer measurements and the BUV data, the standard errors of estimate between their derived ozone values are about .02 atm-cm. The BUV or satellite values appear a little too high or too low depending upon whether the total ozone is high or low respectively. The true standard error of this satellite data is thought to be about 0.015 atm-cm taking into consideration a lack of perfect simultaneity in space and time between the Dobson and satellite data.

Total amounts and vertical distributions of atmospheric ozone data began to be acquired with the launch of Nimbus-4 on April 8, 1970. With the sun under daily surveillance, the launch in April 1974 of Nimbus-4 has made possible simultaneous measurements of the global distribution of atmospheric ozone in the BUV experiments and the ultraviolet solar irradiance by the Monitor of Ultraviolet Solar Energy (MUSE) experiments. There has been nearly five years of average of the temporal behavior of the solar ultraviolet irradiance. Also simultaneously, the interplanetary solar magnetic field has been monitored from space. Heath [10] believes that this data combined should provide a very valuable data base for the development of a climatology of atmospheric ozone.

One limitation on the determination of ozone from the BUV observations discussed by Cunnold, Gray, and Merritt [6] is aerosols. If the concentrations of aerosols at high altitudes are significant, then it is not always possible to separate the effect of the aerosols from those attributed to ozone from observations of backscattered ultraviolet radiation. The presence of aerosols primarily serve to increase the backscattered radiance and therefore lead to the inference of ozone densities that are too low.

### III. Descriptive Statistics

Analyses of total-ozone data have been done on a daily, monthly, seasonal and annual basis as well as longer periods of time looking for latitudinal, longitudinal, short-term and long-term variations or trends.

Wu [26] does an analysis of monthly, seasonal, and annual mean values of total ozone over the Northern Hemisphere for an eleven year period from January, 1960, to December, 1970, and presents the analysis including 17 hemispheric charts. Due to insufficient data, a similar analysis cannot be done in the Southern Hemisphere, but some satellite data is included here in an attempt to grasp the larger picture. Wu's basic source of total-ozone data is "Ozone Data for the World". The total-ozone data is composed of daily measurements from selected stations (except when weather conditions prevent measurements).

From Wu's charts, monthly differences in total-ozone amount are evident. There are important features which indicate seasonal variation. In winter all highs appear to be located on the continents (but there is little data available over the oceans). In the Northern Hemisphere, the ozone content increases in general from winter to spring. In summer there is a big pattern change. Overall, the total-ozone content in the Northern Hemisphere decreases from spring to summer, and of all seasons, fall has the lowest overall total ozone. There is also an annual distribution pattern. The gradient of ozone is smallest in regions of low latitude, then the general trend is an increase from the equator toward the higher mid-latitudes and then a decrease toward the pole. There is one well-formed maximum over the center of the northern part of North America.

In summary for the Northern Hemisphere, the monthly mean total-ozone content varies from about 0.200 to about 0.550 cm depending on geographical location and time of year. The zonally averaged value is smallest over the equatorial belt where the seasonal changes are also small. From  $10^{\circ}$  to  $30^{\circ}$  N the maximum occurs in April or May and the minimum occurs in September or October. As the latitude increases, the monthly mean total-ozone content increases. The amplitude of the seasonal variation increases steadily with latitude towards the pole. From  $30^{\circ}$  N to  $67^{\circ}$  N a mean high total-ozone content is from January to April, highest in April, and a mean low total-ozone content occurs from July to October with the minimum in September. Wu states that the present analysis suggests that planetary waves have the most influence in monthly mean total ozone distributions over the Northern Hemisphere.

It is difficult to evaluate inter-hemispheric differences in the total ozone distribution because the number of observations in the Southern Hemisphere is comparatively few. Wu completes the picture by bringing in recent satellite measurements. Summarizing globally, total ozone at all latitudes exhibits approximately sinusoidal variation with time of year, the maximum values appearing in the spring and the minimum values appearing in autumn. In the tropics, the seasonal changes are minor. In general, the magnitude of total ozone in the Northern Hemisphere increases from the equator to a maximum at about  $60^{\circ}$  -  $70^{\circ}$  N during most of the year and some data suggest that in the spring the maximum is nearer the pole. Patterns of total ozone are different in the two hemispheres. Seasonal and latitudinal variations are similar between the equator and latitude  $55^{\circ}$ , but the ozone content in the middle latitudes of the Southern Hemisphere is higher with a smaller maximum in the spring. Therefore, the mean values over each hemisphere

are nearly the same. Also, total ozone has important longitudinal variations. The largest values of total ozone at all seasons are over northern North America, eastern Asia, and central Europe.

Prabhakara, Rodgers, and Salomonson [24] also describe the seasonal and latitudinal variations in total ozone. They derive the global distribution of total ozone, which can be measured on a daily basis, using data from IRIS, flown on Nimbus 3 and 4 satellites, for the period of April, May, June and July of 1969. Plotting latitudinal averages for the four individual months shows an equatorial minimum in total ozone for all months and an increase of total ozone towards both poles. The total ozone seems to be decreasing with latitude from April to July over the Northern Hemisphere and the reverse is seen in the Southern Hemisphere. Also, the curves suggest that in the Southern Hemisphere, at all latitudes, there is a uniform increase of total ozone from April to July, 1969, but in the Northern Hemisphere, the monthly decrease of total ozone for the same period appears to be nonuniform. This is thought to be due to the difference of land and ocean configuration of the two hemispheres.

Monthly mean maps obtained from the global distribution of total ozone on a daily basis show the isolines of total ozone nearly parallel to latitude circles with the total-ozone amount increasing away from the equator toward both poles.

A comparison of the IRIS total ozone maps with corresponding upper air maps indicates that the total-ozone patterns resemble the distribution of upper air geopotential heights in the upper troposphere. Also, the systematic variation in the total ozone distribution during this period in the tropical, middle and polar latitudes observed indicates changes in the lower stratospheric circulation. This study demonstrated that useful meteorological

information regarding the upper tropospheric flow over data void areas of the world can be obtained from satellite total-ozone measurements.

London and Kelley [17] also give an analysis of the global distribution of total ozone. Mean monthly maps of the total-ozone distribution for each hemisphere covering the 13 year period from 1957 through 1970 were prepared, and grid point values of total ozone for each  $10^\circ$  of latitude and  $20^\circ$  of longitude were extracted to provide data for the analysis. A plot is given of the annual distribution of total ozone over the globe averaged over the entire 13 years. They found the same latitudinal and seasonal variations as Wu. Also, evident geographical variations are more pronounced in the Northern Hemisphere than the Southern Hemisphere perhaps due to the difference in the number of observing stations between the two hemispheres.

It has been observed that there is general agreement between total ozone variations derived from ground-based data and those recently derived from satellite observations for both hemispheres. It is believed that hemispheric differences in the amount and distribution of total ozone result, most probably, from the differences in stratospheric circulation patterns between the two hemispheres.

Kerr [15] conducted an analysis to observe short-term period fluctuations and ozone disturbances. At Toronto, Canada, during the period from September, 1971, to April, 1972, ozone measurements were taken with the Toronto ozone spectrophotometer. It was found that short-lived variations of total ozone occur and sometimes amount to more than 0.1 cm change. Kerr concluded that the ozone disturbance, which can be observed both on the zenith blue sky and on the direct sun, cannot be produced by solar phenomena. This raised many questions as to how and why these disturbances are formed. Also emphasized was the fact that these ozone disturbances may affect daily observations smoothed to obtain means and this is passed on to later analyses.

Heath [10] describes recent advances in satellite observations of solar variability and global atmospheric ozone including long- and short-term as well as seasonal changes related to the unusual stratospheric warming. Acquisition of global atmospheric ozone data from satellites has made it possible to study the hemispheric behavior of total ozone from daily latitudinal zonal means from  $80^{\circ}$  N to  $80^{\circ}$  S which are averaged in longitude over one month intervals. This time averaging is used to minimize possible solar effects related to the 27 day solar rotational period. From satellite observations, a principal goal is to show some of the global changes in ozone in terms of season, solar variability and major stratospheric disturbances such as stratospheric warmings.

Two types of ultraviolet solar variability of interest here are according to their time scales, the eleven year sunspot cycle and the 27 day solar rotational period. There has been evidence of significantly lower ultraviolet solar fluxes in the atomic oxygen production region of the stratosphere, and it has been shown that these solar flux changes occur between the solar maximum and minimum. Therefore it is clear that the sun is an ultraviolet variable star. It has also been shown that there is a significant correlation between enhancements in the ultraviolet solar irradiances and terrestrial passages of the solar magnetic field sector boundary structure.

It is also believed that the global distribution of atmospheric ozone is important in the inference of transport and circulation in the stratosphere. Tabulated on a daily and monthly basis are zonal means of ozone which were averaged in longitude over  $10^{\circ}$  latitude bands. Plots of these seem to indicate a somewhat distorted representation of the temporal change of total ozone with latitude from what one would expect for total ozone



averaged over several years due to a major stratospheric warming that occurred in the Northern Hemisphere in early January, 1971. From observing the latitudinal distributions of the monthly zonal means of total ozone, it has been observed that the seasonal variation of ozone is greatest in the polar regions of the Northern Hemisphere and least for tropical latitudes.

The change of the yearly mean of total ozone with latitude is plotted and shows an asymmetric nature. It has been observed, in comparison, that the mean amounts of ozone from the BUV 1970-1971 data for the Northern Hemisphere maximum, the equatorial minimum, and the Southern Hemisphere maximum are 18%, 14%, and 13% higher respectively than the Dobson 1957-1966 mean data. This indicates that even the yearly mean latitudinal ozone distribution may be enhanced by a major stratospheric warming at high latitudes.

A hemispherical and global ozone budget has been derived from the monthly average of the daily zonal means. To determine the temporal variation of the hemispherical ozone one observes meridional transport, photochemical reactions and destruction mechanisms in the troposphere and at the surface of the earth. The periods of May-June and November-December show the maximum hemispheric ozone gradients with time, but there is a discrepancy as to the rate of loss and increase in the Southern Hemisphere and Northern Hemisphere and whether or not they are balanced changes. There is a significant increase in the magnitude of the short period variability of ozone in the fall-winter period for latitudes of  $20^{\circ}$  N to  $80^{\circ}$  N and  $20^{\circ}$  S to  $80^{\circ}$  S, but in general, total ozone in the Southern Hemisphere can be characterized as being significantly more variable on a short-term basis of days than it is in the Northern Hemisphere.

The stratospheric warming has had some effect. It appears to be associated with the total ozone reaching a maximum at  $60^{\circ}$  N and  $70^{\circ}$  N in

January instead of during the normal March-April period. Here, abnormally high mass mixing ratios were observed at 70° N and are highly suggestive of a non-photochemical source.

#### IV. Inferential Statistics

One aspect of the nature of ozone that makes it difficult to study is the great variability of ozone concentration. It varies considerably as a function of time within a given day, season, latitude, and altitude. There are current physical models for ozone but they do not adequately describe the distribution of ozone nor adequately predict changes in the level of ozone.

Ozone's variation needs to be explained with respect to time and latitude as well as the correlation of ozone concentration with other stratospheric constituents or other factors that may affect ozone. What type of effects make up an ozone measurement? Aside from randomness, some effects that have been considered are seasonal effects, cyclical effects related to the sun, or the possibility of trends in the data.

One approach that has been used in analyzing total-ozone data is trend analysis. In general, a trend analysis is a linear regression analysis where the observation at time  $t$  is a function of time plus a random component. The slope of the regression line represents unit change/unit time, and usually this is converted into percent change per decade. The observations used are the deviations of the monthly means from the monthly normals where the monthly normal is defined as the average for that month taken over all the years for which you have records. The model fitted to these deviations is

$$Y_t = \beta(t) + \epsilon_t$$

deviation of monthly mean from monthly normal      slope      number of months since start of the experiment      error

The  $\epsilon_t$ 's are assumed to be uncorrelated random variables

Pittock [23] used a trend analysis on total ozone data obtained during the first eight years of ozone sounding programs at Aspendale, Victoria, Australia. He estimated a trend of  $(-3.1 \pm 1.1)\%$  per decade for total ozone content. Then a trend analysis was performed on two-month mean total ozone data for days on which there were only ozone soundings to clarify the representativeness of the ozone sounding data. This gave an estimated trend of  $(-2.7 \pm 1.4)\%$  per decade which is in close agreement. Pittock concludes that the observed trends may not be linear but due to the non-linear interaction of the general circulation, chemical composition and photochemistry of the atmosphere.

London and Kelley [17] extracted grid point values of total ozone for each  $10^\circ$  of latitude and  $20^\circ$  of longitude to provide data to derive global total-ozone trends. They observed that there is general agreement between total ozone variations derived from ground-based data and those recently derived from satellite observations for both hemispheres and believed that hemispheric differences in the amount and distribution of total ozone likely result from the differences in stratospheric circulation patterns between the two hemispheres. They used the grid point data to plot hemispheric 3-month running means of the monthly departures of total ozone from the 13-year average for each month using data from 1957 to 1970. For the Northern Hemisphere they had a significant result with the computed trend and probable error for the 13-year change being  $(+7.8 \pm 2.4)\%$  per decade. The trend and

probable error of  $(+2.4 \pm 1.9)\%$  per decade for the Southern Hemisphere was not significant. It appears as if the increase started about March, 1961, in the Northern Hemisphere and about September, 1961, in the Southern Hemisphere but at present the cause of these trends is unknown. Because there was an obvious change in the slope of the trend lines in about 1961, the series of data was divided into two groups based on this apparent change and separate regression lines were constructed. These showed a large increase per decade,  $(11.3 \pm 2.3)\%$ , in the 60's in the Northern Hemisphere but a slight decrease  $(-1.1 \pm 1.6)\%$  per decade in the Southern Hemisphere in the 60's. This seems to indicate that the upward trend in the observed total ozone was a Northern Hemispheric phenomenon. There is still a need to know whether these changes show a definitive world-wide pattern and to determine in what way these trends are associated with fluctuations in stratospheric circulation, changes in solar radiation or general stratospheric photochemistry. Direct solar influence seems to be precluded since there is a definite hemispheric difference in the trends.

Komhyr, Grass, and Slocum [14] use a trend analysis on total ozone from stations in North America in the 1960's. To determine whether an increase in ozone occurred during the 60's, total-ozone data from ten North American stations were analyzed. Plots of mean monthly total-ozone deviations from monthly normals for each station were made for the available periods of record. Then using the method of least squares, ozone trend lines were fitted to the plots. It was determined that the rates of increase of total ozone at all stations varied from  $(2.0 \pm 1.2)\%$  per decade to  $(8.8 \pm 0.7)\%$  per decade. Eight stations had significant trends using the criterion that the sign of the ozone trend is significant if it doesn't change within a range of  $\pm 2$  probable errors. After investigating the

calibration histories of some of the Dobson spectrophotometers, the authors decided that the observed increase in ozone over North America is real and is not largely attributed to calibration drifts nor to observational and data reduction techniques.

Komhyr [13] also reports other data analyses of stations throughout the world. In general, they show increases in total ozone, but these trends are not always statistically significant. Analyses from foreign data as well as North American total ozone data suggest that an increase in total ozone occurred during the past decade over many areas of the world.

Birrer [1], using total ozone data from Arosa, Switzerland, for the 45-year period (1926-1971) does a trend analysis to study atmospheric ozone. He eliminates seasonal changes by using the difference between each mean monthly value of total ozone and the monthly normal. Birrer found that by increasing the length of record by a small amount he could change the sign of the computed trend. To explain why this can occur, he plotted the monthly deviations and found that they follow a quasiperiodic course with varying amplitude. Due to this variability it is difficult to describe an increase or decrease in the world-wide amount of ozone for this periodicity will influence the slope of the trend line.

Trends at all stations would have to be considered to get a world-wide trend value, and it would not be valid to use stations with different lengths of record for this changes trend values. He concludes that only if the record lengths are at least ten years long, then can you use the analysis of all stations in the world to infer a human influence on the atmosphere. As yet though there is not an adequate network of stations over the globe.

Time series analyses and correlations have been used to study total ozone and cyclical effects related to the sun. One of the first studies of

this type was done by Willett [25] using monthly total ozone averages from individual stations. For the 27-year period of 1933-1959 he found a cyclical variation of overall average total ozone. Peak values were reached in 1933, 1941, and 1952, and minimum values were reached in 1937, 1946, and 1955. These three years of peak values occur from zero to three years before the minimum sunspot years of 1933, 1944, and 1954, and the three years of minimum values occur from zero to two years before the sunspot maximum years of 1937, 1947, and 1957. There seems to be a negative correlation of total ozone with sunspots of about 1½ years' lag with respect to the approximately 16-year sunspot cycle. Also, the negative correlation of total ozone with sunspot latitude is found to be almost exactly the same in magnitude to the negative correlation of total ozone and sunspot number. Willett suggests that the phase difference of these correlations indicates that atmospheric ozone may be more sensitive to sunspot latitude than number.

London and Haurwitz [16] give a critical review of Willett's suggestion of a close relationship of sunspot number and total ozone. Some of Willett's data is believed to be incorrect. Observing the number of reporting stations and the months reports were available indicates that the maxima and minima resulted from extreme bias of the data with respect to season and latitude of the reported observations. Recomputing correlations shows a lack of significant relation of total ozone with sunspot data. Grouping all data together each year regardless of season, latitude and longitude cannot give results representative of world-wide total-ozone variations.

London and Oltmans [18] do a time series analysis relating total ozone and sunspots. The statistical analysis is based on the mean annual total ozone at Arosa, Switzerland, and Oxford, U.K., and the relative sunspot

number is used to indicate solar activity. A harmonic filter was applied to the sunspot data to remove a cyclical trend. The now filtered sunspot series was used to compute cross-correlations with the mean annual total-ozone data.

The results showed that evidence for the existence of a significant relationship between mean annual total ozone and annual relative sunspot number disappears in the noise of the data. It is conjectured that variations in total ozone reflect the intervention of related stratospheric and tropospheric circulation processes, which obscures any direct link between ozone and solar variability.

Christie [5] is concerned with secular or cyclic changes in ozone over a longer period of time. Willett did an analysis of total ozone in relation to the sunspot cycle for the period 1933-1957. Here Christie is trying to extend the data sample with data from the World Ozone Data Centre over the period 1958-1971. As is known, the monthly mean relative sunspot number has a distinctive 10-11 year periodicity which is shown in a plot of an autocorrelation function from a time series. Also plotted is the unweighted average total ozone which as one expects shows greatest cyclic variation in its annual cycle, but a plot of the envelopes enclosing the extremes of the seasonal variations indicates a more slowly varying periodicity corresponding to the solar cycle.

After filtering the annual cycle from the ozone data by averaging individual values for a specific month over the period of record and subtracting from individual monthly values to get the ozone residuals, the existence of a solar cycle in these ozone residuals is still evident. There is a significant correlation between the residual of the average total ozone and sunspot number.

Then a rather unsuccessful attempt was made to filter the solar cycle from the ozone residuals by means of a linear regression method to compare predicted values with observed values. A variational trend was still obtained and the sample was considered too restricted to make any conclusion.

Based on the time series of selected stations, it was not felt that the assertion that ozone has increased over the past decade had been conclusively demonstrated since it could be argued that (a) the sample did not adequately represent a global average, (b) periodic variations had not been adequately considered. Suggestions of possible factors for the deviations were aircraft exhaust, emissions, solar flux of ultraviolet, and year-to-year variability in efficiency of transport of ozone through the troposphere to the ozone sink. It was concluded that because of the magnitude of the long-term cycle variations and the large fluctuations in the residuals, the significance of the apparent increasing trend in total ozone in the past decade (i.e. the 60's) appears to be exaggerated.

One aspect involved in a good analysis of ozone data is an adequate network system for obtaining accurate ozone measurements. Pittock [23] questioned the optimal number and location of monitoring stations so that the addition of more monitoring stations would not limit the increase in information by spatial correlations of the data. Dziewulska-Losiowa [7] is concerned with the optimum distribution of ozone stations and uses the method of empirical orthogonal functions to determine an optimum pattern. Bojkov [3] uses a spectral analysis to estimate the optimum distances between measuring stations in longitudinal and latitudinal directions.



## References

1. Birrer, W.M., "Some Critical Remarks on Trend Analysis of Total Ozone Data," Pure and Applied Geophysics (PAGEOPH), Vol. 112, 1974, pp. 523-532.
2. Bojkov, Rumen D., "Some Characteristics of the Total Ozone Deduced from Dobson-spectrophotometer and Filter-ozonometer Data and Their Application to a Determination of the Effectiveness of the Ozone Station Network," Ann. Geophys., Vol. 25, 1969, pp. 293-299.
3. Bojkov, Rumen D., "Planetary Features of the Total and Vertical Ozone Distribution During IQSY," Időjárás, Vol 3-4, 1968, pp. 140-233.
4. Brewer, A.W., and Kerr, J.B., "Total Ozone Measurements in Cloudy Weather," PAGEOPH, Vol 106-108, 1973, pp. 928-937.
5. Christie, A.D., "Secular or Cyclic Change in Ozone," PAGEOPH, Vol. 106-108, 1973, pp. 1000-1009.
6. Cunnold, D.M., Gray, C.R., and Merritt, D.C., "Aerosols: A Limitation on the Determination of Ozone from BUUV Observations," PAGEOPH, Vol. 106-108, 1973, pp. 1264-1271.
7. Dzierwulska-Losiowa, A., "Some Remarks Concerning the International Ozone Network," PAGEOPH, Vol. 106-108, 1973, pp. 971-976.
8. Ferri, Antonio, "Stratospheric Ozone--The Questions Drawn but Not Answered," Astronautics and Aeronautics
9. Heath, Donald F., Mateer, Carlton L., and Krueger, Arlin J., "The Nimbus-4 Backscatter Ultraviolet (BUV) Atmospheric Ozone Experiment--Two Year's Operation," PAGEOPH, Vol. 106-108, 1973, pp. 1238-1253.
10. Heath, Donald F., "Recent Advances in Satellite Observations of Solar Variability and Global Atmospheric Ozone," Preprint, June, 1974.

11. Haring, Wayne S., and Borden, Thomas R., Jr., "Ozonesonde Observations over North America," Vol. 1-2, AFCRL-64-30 (11), July, 1964.
12. Kerr, J.B., "Short-time Period Fluctuations in the Total Ozone," PAGEOPH, Vol. 106-108, 1973, pp. 977-980.
13. Komhyr, W.D., Barrett, E.W., Slocum, G., and Weickmann, H.K., "Atmospheric Total Ozone Increase during the 1960's," Nature, Vol. 232, 1971, pp. 390-391.
14. Komhyr, W.D., Grass, R.D., Slocum, G., "Total Ozone Increase over North America During the 1960's," PAGEOPH, Vol. 106-108, 1973, pp. 981-999.
15. Krueger, A.J., and Pressman, J., "VIII Ozone Observations," Revised Draft, July, 1973.
16. London, Julius, and Haurwitz, Marion W., "Ozone and Sunspots," Journal of Geophysical Research, Vol. 68(3), 1963, pp. 795-801.
17. London, Julius, and Kelley, Jean, "Global Trends in Total Atmospheric Ozone," Science, Vol. 184, 1974, pp. 987-989.
18. London, Julius and Oltmans, Samuel, "Further Studies of Ozone and Sunspots," PAGEOPH, Vol. 106-108, 1973, pp. 1302-1307.
19. Mani, A., "Problems of Standardization of Total Ozone Measurements," PAGEOPH, Vol. 106-108, 1973, pp. 967-970.
20. Mateer, Carlton L, Heath, Donald F., and Krueger, Arlin J., "Estimation of Total Ozone from Satellite Measurements of Backscattered Ultra-violet Earth Radiance," Journal of the Atmos. Sci., Vol. 28, 1971, pp. 1307-1311.
21. Ozone Data for the World, Canadian Meteorological Service in Cooperation with the World Meteorological Organization, Toronto, Ontario, Oct., 1970.

22. Picha, J., "Total Ozone and Temperature Field at 100 mb over Europe 1967-1968," PAGEOPH, Vol. 106-108, 1973, pp. 962-966.
23. Pittcock, A.B., "Ozone Climatology, Trends and the Monitoring Problem," Conference on Structure, Composition and General Circulation of the Upper and Lower Atmospheres and Possible Anthropogenic Perturbations, Vol. 1, 1974, pp. 455-466.
24. Prabhakara, C., Rodgers, E.B., and Salomonson, V.V., "Remote Sensing of the Global Distribution of Total Ozone and the Inferred Upper-Tropospheric Circulation from Nimbus IRIS Experiments," PAGEOPH, Vol. 106-108, 1973, pp. 1226-1237.
25. Willett, Hurd C., "The Relationship of Total Atmospheric Ozone to the Sunspot Cycle," Journal of Geophysical Research, Vol. 67, 1962, pp. 661-670.
26. Wu, Mao-Fou, "Observation and Analysis of Trace Constituents in the Stratosphere," National Technical Information Service, Dept. of Commerce, PB 231 958, 1973.
27. Urbach, Frederick; Berger, Daniel, and Davies, Ronald E., "Field Measurements of Biologically Effective UV Radiation and its Relation to Skin Cancer in Man," Proceedings of the Third Conference on the Climatic Impact Assessment Program (CIAP), DOT-TSC-OST-74-15, 1974, pp. 523-535.

# An Evaluation of Trend Analysis as Used on Ozone Data

Many authors in recent years have used trend analysis procedures to determine if there has been an increase or decrease in the amount of ozone. (E.g. Birrer [1974], Komhyr [1971], Komhyr [1973], London [1974], Pittock [1974].) We would like to suggest, however, that there is a serious flaw in the way this procedure is being applied to ozone data.

In theory a trend analysis applicable when one can assume that the data at time  $t$  can be decomposed into a polynomial in  $t$  and random error or noise. I.e. an appropriate model is

$$Y_t = f(t) + \epsilon_t$$

observation at time  $t$       polynomial in  $t$       error at time  $t$

The errors,  $\epsilon_t$ 's, are uncorrelated random variables with mean zero and variance  $\sigma^2$ . With this model and these assumptions one can use the least squares theory to find estimators for the components of  $f(t)$ . With the additional assumption that the errors are normally distributed one can test to see if one of the components is equal to zero. For example suppose this model is fitted--

$$Y_t = \mu + \beta(t) + \epsilon_t$$

observation at time  $t$       intercept      slope      time since start of experiment      error

Then one can use the usual least squares theory to find the estimator  $\hat{\beta}$  and  $\text{Var}(\hat{\beta})$ . If one assumes the errors are normal, then one can test to see if  $\hat{\beta}$ , the slope, is zero. (This is what you are doing when you check to see if lies within two standard deviations of zero.)

Ozone clearly has a seasonal component as well as a noise component and a possible trend component. To remove this seasonal effect from the data many authors have used monthly deviations from monthly normals instead of monthly means. To explain what this is, suppose one has data from 1960-1969. Then the January, 1965, deviation is the mean for January, 1965, minus the overall mean of all January means from 1960-1969. The seasonal effect due to being in the month of January is removed from the data by this operation. The authors cited above then fit the following model and check to see if the slope is nonzero.

$$Y_{M,Y} = \beta(12Y + M) + \epsilon_{M,Y}$$

$\uparrow$  slope       $\uparrow$  error

deviation for month M of year Y since start of the experiment      number of months since the start of the experiment

$$M = 1, 2, \dots, 12$$

$$Y = 0, 1, \dots, n-1$$

$n$  = number of years of data available

The usual assumptions are made that the errors are independently identically distributed.

We would like to show here that this is not the correct model to fit. To see this, one must look more closely at what occurs when one takes monthly deviations from monthly normals. Let's represent the mean ozone level for month M of year Y since the start of the experiment or  $Z_{M,Y}$  as

$$Z_{M,Y} = \text{Seasonal}(M) + \beta(12Y + M) + \epsilon_{M,Y}$$

Seasonal effect due to being in the mth month  
 unit change in ozone per month  
 number of months since start of the experiment  
 error

Then the monthly normal for month M would be the average of all the  $Z_{M,Y}$ 's taken over all possible values of Y. That is we can represent the monthly normal for month M by

$$\bar{Z}_{M.} = \frac{1}{n} \sum_{Y=0}^{n-1} Z_{M,Y} = \frac{1}{n} \sum_{Y=0}^{n-1} [\text{Seasonal}(M) + \beta(12Y + M) + \epsilon_{M,Y}]$$

$$\bar{Z}_{M.} = \text{Seasonal}(M) + \beta(12\bar{Y} + M) + \bar{\epsilon}_{M.}$$

$$\text{where } \bar{Y} = \frac{1}{n} \sum_{Y=0}^{n-1} (Y) = \frac{n-1}{2} \quad \text{and} \quad \bar{\epsilon}_{M.} = \frac{1}{n} \sum_{Y=0}^{n-1} \epsilon_{M,Y}$$

The deviation of month M of year Y is simply the difference between these two or

$$Y_{M,Y} = Z_{M,Y} - \bar{Z}_M = [\text{Seasonal}(M) + \beta(12Y + M) + \epsilon_{M,Y}] - [\text{Seasonal}(M) + \beta(12\bar{Y} + M) + \bar{\epsilon}_M]$$

or

$$y_{M,Y} = \beta(12Y - 12\bar{Y}) + e_{M,Y}$$

where  $Y_{M,Y}$  is the deviation of Mth month of Yth year from the overall average for the Mth month and  $e_{M,Y} = \epsilon_{M,Y} - \bar{\epsilon}_M$ .

Notice: The month  $M$  of the year does not appear anywhere in the model representing the deviation of the  $M$ th month of year  $Y$  from the  $M$ th monthly normal. This implies that in considering these monthly deviations it is not important what month one is in but only the number of years  $Y$  since the start of the experiment. Hence each monthly deviation from monthly normal for year  $Y'$  is measuring the same quantity  $\beta[12Y' - 12\bar{Y}]$  apart from error. Therefore if one fits a model regressing the monthly deviations on the number of months since the start of the experiment or  $(12Y + M)$  the  $\beta$  one obtains will not be a measure of the amount of increase of ozone per month. If one wanted to use the trend analysis procedure on monthly deviations from monthly normals, then one should fit the model:

$$Y_{M,Y} = \beta(12Y - 12\bar{Y}) + e_{M,Y}$$

and then one can interpret the  $\hat{\beta}$  obtained as an estimate of the amount of increase of ozone per month. This model is equivalent to

$$Y_{M,Y} = \mu + \beta(Y) + e_{M,Y}$$

If this is the model that is fitted there still are some objections to the procedure. Five items that need to be considered are--

(i) Is it reasonable to assume the errors are uncorrelated? This is an essential requirement for a trend analysis.

(ii) The rule of thumb for deciding if  $\beta$  is zero is to see if its estimator,  $\hat{\beta}$ , lies within two standard deviations of zero. This is based upon the tacit assumption that the errors are normally distributed.

(iii) A common practice is to convert the estimated slope,  $\hat{\beta}$ , from change in ozone/unit of time to percentage change in ozone/unit of time. To do this one needs to divide  $\hat{\beta}$  by some function of the data representing

the total amount of change. ( $\hat{\beta}' = \hat{\beta}/T$ ). Since  $T$  is a function of the data one cannot easily find  $\text{Var } \hat{\beta}'$  and it is not equal to  $\frac{1}{T} \text{Var } \hat{\beta}$ . If  $\frac{1}{T} \text{Var } \hat{\beta}$  is used as an approximation for the variance, one should be aware that its bias is not known and hence it may be unreliable.

(iv) If there is a long range cyclical effect in ozone observations due to say, the sunspot cycle, then the trend one obtains will reflect the cyclical effect as well as any trend effect that may exist. In this case you cannot determine if a significant trend is the result of a real trend in the data or simply due to being in a certain phase of the cycle.

All four of these considerations apply to the model (1) fitted by the authors cited. But model (1) we have shown to be incorrect. These four also apply to model (2), the correct model, as well as this criticism.

(v) In model (2) one would expect the within year variation to be much larger than the between year variation so that the trend effect would be obscured by noise. (2), although the correct model, is not a satisfactory model for this reason.

Our final conclusion is that trend analysis has been misused in analyzing ozone data. Even when the correct model is used this technique is really not suitable for analyzing ozone data. It's chief fault is that it is based on too simple a model--a model that cannot adequately describe the distribution of ozone. One needs a model that will allow for seasonal effects, cyclical effects, trend effects and correlated errors. This type of model will not force one to make certain shaky assumptions such as the assumption of the trend analysis that the observations are not correlated with respect to time.



## References

1. Anderson, T.W., The Statistical Analysis of Time Series, John Wiley & Sons, Inc., New York, 1971
2. Birrer, W.M., "Some Critical Remarks on Trend Analysis of Total Ozone Data," Pure and Applied Geophysics, Vol. 112, 1974, pp. 523-532.
3. Komhyr, W.S., Barrett, E.W., Slocum, G., and Weickmann, H.K., "Atmospheric Total Ozone Increase During the 1960's," Nature, Vol. 232, 1971, pp. 390-391.
4. Komhyr, W.S., Grass, R.D., Slocum, G., "Total Ozone Increase over North America During the 1960's," Pure and Applied Geophysics, Vol. 106-108 1973, pp. 981-999.
5. London, Julius and Kelley, Jean, "Global Trends in Total Atmospheric Ozone," Science, Vol. 184, 1974, pp. 987-989.
6. Pittock, A.B., "Ozone Climatology, Trends and the Monitoring Problem," Conference on Structure, Composition and General Circulation of the Upper and Lower Atmospheres and Possible Anthropogenic Perturbations, Vol. 1, 1974, pp. 455-466.

## Analyzing Ozone Data

As mentioned previously one of the most striking characteristics of ozone observations are their high variability. There is now no deterministic model that adequately describes the distribution of ozone and there may never be one. For this reason it is clear that what is necessary is a stochastic model that will reflect the natural variability of ozone. A time series stochastic model is most appropriate in this instance since it allows for observations that are time dependent (as ozone observations certainly are). We will be investigating two different time series approaches to analyzing ozone data.

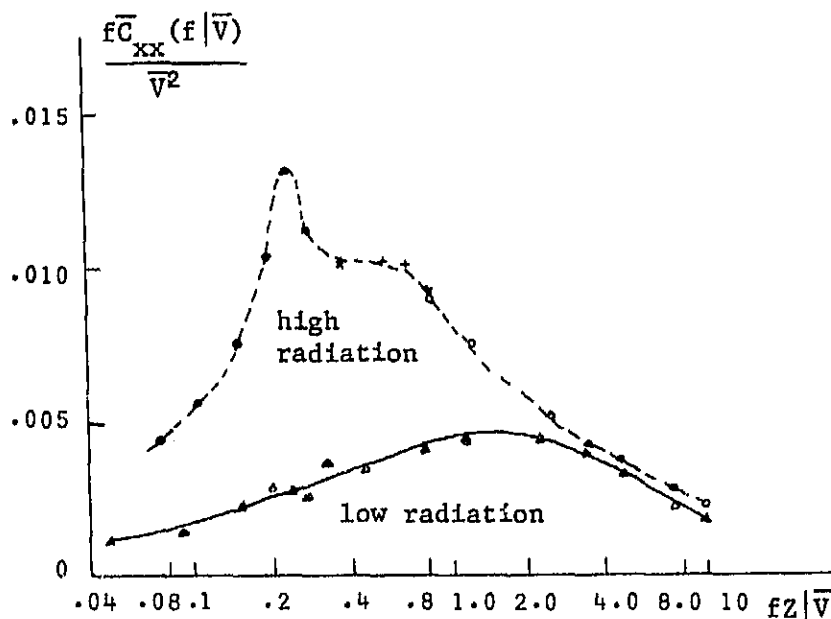
The first approach we will take will involve fitting a parametric model to the data--specifically a seasonal autoregressive integrated moving average model. This model is based upon the idea that observations from a time series can be thought of as the result of white noises or random "shocks" that have been transformed by a linear filter. Two properties of this technique are that optimal forecasts of future values of the time series can be made and more importantly, it allows the development of a transfer function model. For example in considering ozone data we will be able to make forecasts about future values of the series. Of greater utility to those scientists involved in ozone research will be the development of a transfer function model which will describe the inertial characteristics of the system and help us obtain an understanding of the mechanisms generating the system. For example, currently we are involved in modeling the relationship between solar radiation and ozone. As a measure of solar radiation we are using mean monthly sunspot number and comparing it

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to mean monthly ozone level at various stations. As a result of this analysis we will be able to determine if there is a relationship between sunspots and ozone and to describe this relationship. We will be doing this for many stations and can compare to see if there are significant differences between, say, stations in the mid-latitudes and stations in equatorial regions. Connected with the use of this type of time series model we will be investigating two topics. First we will be checking to determine other possible input parameters that may be added to the transfer function models. (A conference between us and some of the scientists investigating ozone will probably be necessary here. We need to know what may affect ozone and where we can obtain data about the input parameters.) Another topic we will be investigating will involve searching for techniques that allow one to determine whether a change in level has occurred in a time series. For example, suppose one has observations from a station that seem to be following a certain pattern and then a freak event happens near the station (e.g. a volcano erupts) and there appears to be a drop in the level of the process. How does one determine if this drop is an actual (perhaps permanent) decline in the level of the system? We plan to search the literature to see if we can find techniques that fit our particular type of data or that can be adapted for our use.

The second type of time series approach we want to investigate is a spectral analysis of the ozone data. Spectral analysis is a nonparametric method that involves splitting the time series into different frequency components. The shape of a spectrum may point up features that need to be explained in modeling the system. The presence of peaks in the spectrum and the heights of these peaks may suggest physical interpretations and aid one in obtaining an understanding of the mechanism generating the time

series. To illustrate what we mean consider the example from Jenkins and Watts [1969] of the spectrum of the horizontal velocity component of atmospheric turbulence. Note that the peak in the spectrum moves toward lower frequencies with increasing radiance and at higher frequencies the power seems to be independent of radiation. This and similar studies suggest the



Velocity Spectra for Horizontal Wind Components

following interpretation--there are two different causes for the fluctuations in atmospheric turbulence; at high frequencies the main cause of atmospheric turbulence is frictional forces and at low frequencies the main cause is heat convection due to solar radiation. Thus we hope that one of the results of a spectral analysis will be a better understanding of the system and hence the construction of a realistic model for ozone should be made easier. Besides this, spectral analysis can also be used to determine the optimal distribution of stations (with respect to latitude and longitude) that will allow one to adequately characterize the distribution of ozone. This infor-

mation is necessary in creating a world-wide ozone monitoring network based on ground based stations and in determining optimal satellite sampling techniques.

Finally in using these two techniques in analyzing ozone data we will be investigating the possibilities of each as far as detecting a linear trend in the data. The techniques will provide us with an understanding of the system that is vitally necessary in finding a real trend. It is of little use to detect a trend in the data if that trend is due to the sun-spot cycle or some other natural phenomena that affect ozone. We want to detect a trend that cannot be explained as part of the natural variability of ozone. If one can describe the natural variability of ozone then the argument that there has or has not been a change due to nuclear testing, aerosol use, or exhaust effluents from SST's will carry weight.

## References

1. Anderson, T.W., The Statistical Analysis of Time Series, John Wiley & Sons, Inc., New York, 1971.
2. Box, George E.P., and Gwilyn M. Jenkins, Time Series Analysis, Holden-Day, San Francisco, 1970.
3. Jenkins, Gwilyn M. and Donald G. Watts, Spectral Analysis and Its Applications, Holden-Day, San Francisco, 1969.
4. Koopmans, L.H., The Spectral Analysis of Time Series, Academic Press, New York, 1974.